MG226 : Advanced Analytics
Midterm 2019

1. Based on a sample of size 30, the stiffness or modulus of elasticity and bending strength of a certain variety of lumber, both measured in thousands of psi (pounds per square-inch), are found to have mean $(2, 8)'$ and (sample) variance-covariance matrix

$$\frac{1}{17}\begin{bmatrix} 1 & -4 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ -4 & 1 \end{bmatrix}.$$

Assuming the two variables jointly to have a bivariate Normal distribution, answer the following:

a. Give 95% confidence intervals for the mean stiffness and bending strength of the lumber variety, independently for the two variables **$[2\frac{1}{2} + 2\frac{1}{2} = 5]$**

b. Analytically express the 95% **joint** confidence region for these two means as a subset of $\Re^2$. Plot this region in a provided graph-paper, with clear marking and mention of the co-ordinates of the points that are minimally necessary for identifying this region in $\Re^2$. **[2+6=8]**

c. Individually, between what ranges of values do these two means lie, according to the 95% joint confidence region obtained in **b**? How and why do these intervals differ from the ones found in **a**? **[2+2+1+2=7]**

2. Let $\begin{pmatrix} X \\ Y \end{pmatrix} \sim N_2\left(\begin{pmatrix} 1 \\ 3 \end{pmatrix}, \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}\right)$. Answer the following:

a. Do $(X, Y)'$ possess a joint p.d.f. in $\Re^2$? Explain why or why not. **[2]**

b. Find constants m and c such that $P(Y = mX + c) = 1$. **[2]**

c. Depict the Normal distribution, as specified in $\Re^2$, by plotting its support, marking its mean and standard deviation in this support, and approximately sketching the density function on this support. **[6]**

3. Let Z_1, Z_2 and Z_3 respectively denote standardised values (mean 0, standard deviation 1) of performance, risk-taking-attitude, and experience of investment bankers. The covariance matrix of $(Z_1, Z_2, Z_3)'$ is $\begin{bmatrix} 1 & -0.35 & 0.82 \\ -0.35 & 1 & -0.60 \\ 0.82 & -0.60 & 1 \end{bmatrix}$. Answer the following:

a. Give a 95% prediction interval for the standardised performance of an investment banker who has a standardised risk-taking-attitude of 1.0 and a standardised experience of -0.5. **[7]**

b. Give a linear combination of Z_1, Z_2 and Z_3 which is independent of $(Z_2, Z_3)'$. **[3]**