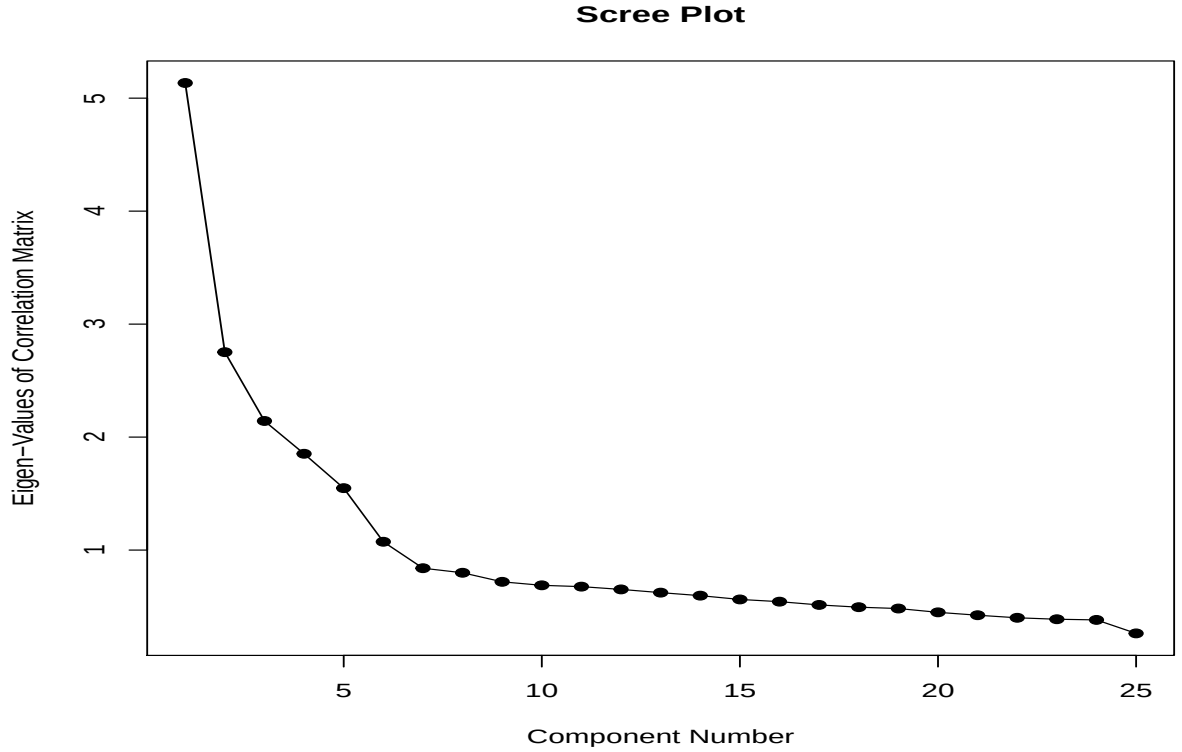
Illustration of Factor Analysis

Example 1: The first example involves the `bfi` data, pertainig to responses to 25 Personality Test Items taken by 2436 (probably neurotic) individuals, which we have already studied for PCA. Recall that these 25 items are supposed to belong to 5 broad dimensions measuring **Agreeability** (items `-A1, A2, A3, A4 & A5`), **Conscientious** (items `C1, C2, C3, -C4 & -C5`), **Extraversion** (items `-E1, -E2, E3, E4 & E5`), **Neuroticism** (items `N1, N2, N3, N4 & N5`) and **Openness** (items `O1, -O2, O3, O4 & -O5`). We begin where we left off in the PCA handout with this data set's PCA.

```
> pca.bfi<-principal(X,nfactors=25)  #library(psych)

> pca.bfi$values
 [1] 5.1343112 2.7518867 2.1427020 1.8523276 1.5481628 1.0735825 0.8395389
 [8] 0.7992062 0.7189892 0.6880888 0.6763734 0.6517998 0.6232530 0.5965628
[15] 0.5630908 0.5433053 0.5145175 0.4945031 0.4826395 0.4489210 0.4233661
[22] 0.4006715 0.3878045 0.3818568 0.2625390

> plot(1:25,pca.bfi$values,pch=19,xlab="Component Number",ylab="Eigen-Values
+ of Correlation Matrix",main="Scree Plot")
> lines(c(1:25),pca2.cor$sd^2)
```



```
> pca.bfi$loadings

Loadings:
   RC2    RC6    RC8    RC7    RC5    RC9    RC10   RC14   RC4    RC3    RC12
A1                          0.973
A2                   0.148 -0.177
A3                   0.165 -0.128
A4                   0.948
A5                   0.125
C1                                 0.131                      0.943
C2                                 0.157                      0.197
C3                                 0.954                      0.127
C4                                -0.155                     -0.154
C5                   -0.101        -0.154
E1          0.941
E2          0.219  0.111
E3         -0.126                                    0.141
E4         -0.187         0.128
E5         -0.123                                    0.120          0.105
N1  0.414          0.163
N2  0.899          0.137
N3  0.247          0.189
N4  0.148  0.101   0.175                      0.102
N5  0.132          0.941
O1                                                   0.951                -0.101
O2                                                          0.965          0.146
O3                                                   0.170 -0.133        -0.139
O4                                                         0.974
O5                                                                0.148          0.962
   RC23   RC13   RC17   RC16   RC11   RC19   RC18   RC20   RC15   RC21   RC22
A1                                   -0.145                      -0.100
A2                    0.111  0.140  0.913                       0.200
A3                           0.209  0.214  0.142                0.894
A4                           0.107  0.133                       0.139
A5                           0.897  0.147  0.154                0.207
C1                0.188                                                 -0.135
C2         -0.110 0.929                                                 -0.160
C3         -0.136 0.145                                                 -0.132
C4          0.209 -0.173                                                 0.905
C5          0.916 -0.115                                          0.128  0.202
E1                       -0.112                -0.111
E2                       -0.147 -0.103         -0.138          0.116
E3                        0.145  0.152          0.903  0.167          0.138
E4                        0.105  0.192          0.154         -0.108  0.133
E5                        0.915          0.111  0.141  0.109
N1  0.249                                                    0.150
N2  0.224                                                    0.132
```

```
N3  0.892                                                       0.218
N4  0.221  0.139                                                0.891
N5  0.163                                                       0.148
O1                       0.106                0.121  0.154
O2                                                   -0.116
O3                       0.106                0.157  0.922
O4
O5                                                   -0.123
   RC24   RC1    RC25
A1
A2
A3  0.125
A4  0.102
A5  0.178
C1
C2
C3
C4
C5
E1 -0.150  0.176
E2 -0.205  0.879
E3  0.141 -0.128
E4  0.876 -0.208
E5        -0.131
N1                0.825
N2                0.263
N3                0.191
N4 -0.104  0.112  0.120
N5                0.111
O1
O2
O3
O4
O5

                 RC2   RC6   RC8   RC7   RC5   RC9  RC10  RC14   RC4   RC3
SS loadings    1.116 1.043 1.039 1.030 1.026 1.022 1.019 1.015 1.014 1.013
Proportion Var 0.045 0.042 0.042 0.041 0.041 0.041 0.041 0.041 0.041 0.041
Cumulative Var 0.045 0.086 0.128 0.169 0.210 0.251 0.292 0.332 0.373 0.413
                RC12  RC23  RC13  RC17  RC16  RC11  RC19  RC18  RC20  RC15
SS loadings    1.012 1.004 0.999 0.999 0.998 0.994 0.993 0.992 0.991 0.982
Proportion Var 0.040 0.040 0.040 0.040 0.040 0.040 0.040 0.040 0.040 0.039
Cumulative Var 0.454 0.494 0.534 0.574 0.614 0.654 0.693 0.733 0.773 0.812
                RC21  RC22  RC24   RC1  RC25
SS loadings    0.977 0.975 0.957 0.957 0.834
Proportion Var 0.039 0.039 0.038 0.038 0.033
Cumulative Var 0.851 0.890 0.928 0.967 1.000
```

```
> fa5.bfi<-factanal(X,5)
> fa5.bfi

Uniquenesses:
   A1    A2    A3    A4    A5    C1    C2    C3    C4    C5    E1    E2    E3
0.830 0.576 0.466 0.691 0.512 0.660 0.569 0.677 0.510 0.557 0.634 0.454 0.558
   E4    E5    N1    N2    N3    N4    N5    O1    O2    O3    O4    O5
0.468 0.592 0.271 0.337 0.478 0.507 0.664 0.675 0.744 0.518 0.752 0.726

Loadings:
   Factor1 Factor2 Factor3 Factor4 Factor5
A1  0.104                  -0.393
A2          0.191   0.144   0.601
A3          0.280   0.110   0.662
A4          0.181   0.234   0.454  -0.109
A5 -0.124   0.351           0.580
C1                  0.533           0.221
C2                  0.624   0.127   0.140
C3                  0.554   0.122
C4  0.218          -0.653
C5  0.272  -0.190  -0.573
E1         -0.587          -0.120
E2  0.233  -0.674  -0.106  -0.151
E3          0.490           0.315   0.313
E4 -0.121   0.613           0.363
E5          0.491   0.310   0.120   0.234
N1  0.816                  -0.214
N2  0.787                  -0.202
N3  0.714
N4  0.562  -0.367  -0.192
N5  0.518  -0.187           0.106  -0.137
O1          0.182   0.103           0.524
O2  0.163          -0.113   0.102  -0.454
O3          0.276           0.153   0.614
O4  0.207  -0.220           0.144   0.368
O5                                 -0.512

                Factor1 Factor2 Factor3 Factor4 Factor5
SS loadings       2.687   2.320   2.034   1.978   1.557
Proportion Var    0.107   0.093   0.081   0.079   0.062
Cumulative Var    0.107   0.200   0.282   0.361   0.423

Test of the hypothesis that 5 factors are sufficient.
The chi square statistic is 1490.59 on 185 degrees of freedom.
The p-value is 1.22e-202
```

```
> fa6.bfi<-factanal(X,6)
> fa6.bfi
Call:
factanal(x = X, factors = 6)

Uniquenesses:
   A1    A2    A3    A4    A5    C1    C2    C3    C4    C5    E1    E2    E3
0.675 0.482 0.473 0.696 0.516 0.637 0.498 0.685 0.424 0.566 0.613 0.453 0.522
   E4    E5    N1    N2    N3    N4    N5    O1    O2    O3    O4    O5
0.432 0.599 0.273 0.302 0.480 0.504 0.652 0.663 0.703 0.510 0.756 0.640

Loadings:
   Factor1 Factor2 Factor3 Factor4 Factor5 Factor6
A1  0.106                  -0.530  -0.105   0.135
A2          0.239   0.128   0.663
A3          0.359   0.122   0.601           0.143
A4          0.231   0.243   0.401  -0.129
A5 -0.142   0.435   0.107   0.461           0.224
C1                  0.557           0.189
C2                  0.677                   0.160
C3                  0.548   0.107
C4  0.219          -0.638  -0.105  -0.133   0.300
C5  0.278  -0.182  -0.546                   0.143
E1         -0.583          -0.120           0.156
E2  0.239  -0.674          -0.110           0.114
E3          0.566   0.102   0.170   0.251   0.237
E4 -0.136   0.674   0.115   0.222  -0.107   0.148
E5          0.506   0.303           0.207
N1  0.819                  -0.167          -0.125
N2  0.805                  -0.128          -0.176
N3  0.710
N4  0.561  -0.344  -0.162                   0.181
N5  0.512  -0.163                  -0.158   0.165
O1          0.234   0.127           0.483   0.181
O2  0.158                          -0.494   0.137
O3          0.337                   0.572   0.193
O4  0.205  -0.169           0.133   0.352   0.177
O5                                 -0.572   0.146

                Factor1 Factor2 Factor3 Factor4 Factor5 Factor6
SS loadings       2.722   2.652   2.075   1.672   1.512   0.614
Proportion Var    0.109   0.106   0.083   0.067   0.060   0.025
Cumulative Var    0.109   0.215   0.298   0.365   0.425   0.450

Test of the hypothesis that 6 factors are sufficient.
The chi square statistic is 896.7 on 165 degrees of freedom.
The p-value is 5.73e-101
```

```
> factanal(X,12)$PVAL
0.008732393
> factanal(X,13)$PVAL
0.07265011
> factanal(X,14)$PVAL
0.2868025

> factanal(X,13)

Call:
factanal(x = X, factors = 14)

Uniquenesses:
   A1    A2    A3    A4    A5    C1    C2    C3    C4    C5    E1    E2    E3
0.694 0.357 0.373 0.005 0.489 0.590 0.447 0.670 0.382 0.265 0.494 0.388 0.425
   E4    E5    N1    N2    N3    N4    N5    O1    O2    O3    O4    O5
0.307 0.503 0.255 0.257 0.397 0.409 0.465 0.638 0.666 0.457 0.720 0.626

Loadings:
   Factor1 Factor2 Factor3 Factor4 Factor5 Factor6 Factor7 Factor8 Factor9
A1  0.110                                   -0.519
A2          0.186   0.150           0.203   0.692   0.124
A3          0.156   0.111           0.560   0.481   0.153
A4          0.125   0.151           0.176   0.220   0.928
A5 -0.156   0.269   0.110           0.539   0.300   0.112
C1                  0.600  -0.166
C2                  0.707           0.102           0.123
C3                  0.526                   0.104          -0.159
C4  0.166  -0.127  -0.600   0.171          -0.130           0.237   0.157
C5  0.224  -0.122  -0.431                                   0.682
E1         -0.670           0.100          -0.131
E2  0.178  -0.682                  -0.181                   0.118   0.160
E3          0.361          -0.225   0.558
E4 -0.120   0.621   0.129           0.412           0.128
E5          0.398   0.320  -0.160   0.190                           -0.147
N1  0.833                                   -0.125
N2  0.830
N3  0.711                                                           0.139
N4  0.526  -0.319  -0.108                                   0.177   0.186
N5  0.463  -0.110           0.181                                   0.510
O1                  0.134  -0.445   0.238
O2  0.113                   0.538
O3          0.200   0.114  -0.579   0.299                           0.113
O4  0.165  -0.215          -0.302           0.112           0.130   0.100
O5                          0.582          -0.108
   Factor10 Factor11 Factor12 Factor13
```

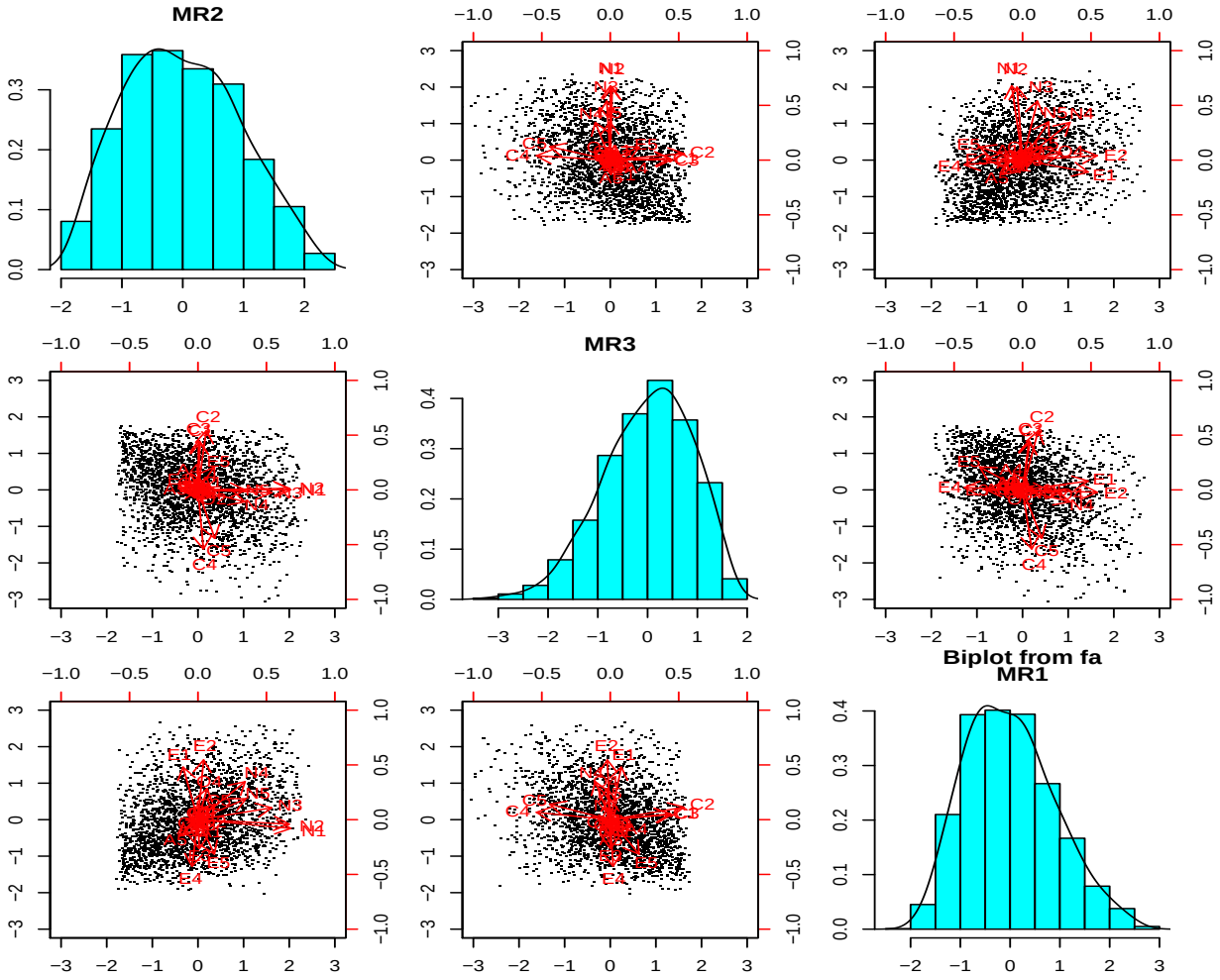
```
A1
A2  0.199
A3
A4
A5
C1
C2
C3  0.100
C4  0.147                       0.225
C5
E1
E2
E3  0.232                      -0.105
E4                    0.198     0.172
E5  0.370
N1
N2          -0.169
N3           0.241
N4           0.354
N5
O1  0.259
O2
O3  0.155                       0.150
O4                    0.251
O5                              0.101

                Factor1 Factor2 Factor3 Factor4 Factor5 Factor6 Factor7 Factor8
SS loadings       2.594   1.982   1.944   1.473   1.432   1.235   0.973   0.641
Proportion Var    0.104   0.079   0.078   0.059   0.057   0.049   0.039   0.026
Cumulative Var    0.104   0.183   0.261   0.320   0.377   0.426   0.465   0.491
                Factor9 Factor10 Factor11 Factor12 Factor13
SS loadings       0.448    0.397    0.257    0.189    0.155
Proportion Var    0.018    0.016    0.010    0.008    0.006
Cumulative Var    0.509    0.525    0.535    0.543    0.549

Test of the hypothesis that 13 factors are sufficient.
The chi square statistic is 68.66 on 53 degrees of freedom.
The p-value is 0.0727


> factanal(X,14)

Call:
factanal(x = X, factors = 14)

Uniquenesses:
   A1    A2    A3    A4    A5    C1    C2    C3    C4    C5    E1    E2    E3
```

```
0.719 0.251 0.290 0.005 0.519 0.595 0.465 0.670 0.380 0.316 0.450 0.432 0.396
   E4    E5    N1    N2    N3    N4    N5    O1    O2    O3    O4    O5
0.163 0.516 0.278 0.217 0.396 0.417 0.452 0.648 0.645 0.478 0.005 0.634

Loadings:
   Factor1 Factor2 Factor3 Factor4 Factor5 Factor6 Factor7 Factor8 Factor9
A1  0.116                  -0.478           0.116
A2          0.152  -0.144   0.784   0.180                   0.109
A3          0.109  -0.116   0.578   0.347                   0.159   0.135
A4          0.154           0.250                           0.931
A5 -0.169   0.104  -0.200   0.382   0.379                   0.126   0.246
C1          0.599                           -0.136
C2          0.698                   0.133                   0.123
C3          0.532           0.108
C4  0.163  -0.605   0.141  -0.105   0.121   0.231
C5  0.226  -0.445   0.103                                  -0.103
E1                  0.703  -0.135  -0.132
E2  0.182           0.604  -0.117  -0.234           0.104          -0.156
E3                 -0.294   0.114   0.677
E4 -0.125   0.117  -0.493   0.149   0.280   0.123           0.127   0.654
E5          0.329  -0.367   0.109   0.380
N1  0.821                  -0.130
N2  0.847
N3  0.707
N4  0.526  -0.114   0.305
N5  0.453                                   0.192
O1          0.138                   0.424  -0.344
O2  0.103                                   0.573
O3          0.117  -0.159           0.462  -0.468
O4  0.131           0.127                  -0.161   0.956
O5                                         0.571
   Factor10 Factor11 Factor12 Factor13 Factor14
A1
A2                   -0.175             0.105
A3                    0.422
A4
A5                    0.168
C1
C2
C3 -0.156
C4  0.227    0.124                      0.235
C5  0.629
E1
E2  0.125    0.171
E3                                    -0.112
E4
E5          -0.207                      0.122
```

```
N1
N2                          -0.204
N3          0.154            0.249
N4  0.181   0.200            0.329
N5          0.532
O1                                    0.114
O2
O3                                    0.147
O4
O5

                Factor1 Factor2 Factor3 Factor4 Factor5 Factor6 Factor7 Factor8
SS loadings       2.584   1.961   1.594   1.532   1.525   1.208   0.988   0.986
Proportion Var    0.103   0.078   0.064   0.061   0.061   0.048   0.040   0.039
Cumulative Var    0.103   0.182   0.246   0.307   0.368   0.416   0.456   0.495
                Factor9 Factor10 Factor11 Factor12 Factor13 Factor14
SS loadings       0.574    0.559    0.478    0.267    0.247    0.161
Proportion Var    0.023    0.022    0.019    0.011    0.010    0.006
Cumulative Var    0.518    0.540    0.560    0.570    0.580    0.586

Test of the hypothesis that 14 factors are sufficient.
The chi square statistic is 45.6 on 41 degrees of freedom.
The p-value is 0.287

> fa(X,6)                                                 # library(psych)
Loading required namespace: GPArotation

Factor Analysis using method =  minres
Call: fa(r = X, nfactors = 6)

Standardized loadings (pattern matrix) based upon correlation matrix
     MR2   MR3   MR1   MR5   MR4   MR6   h2   u2 com
A1  0.09  0.07 -0.11 -0.56  0.03  0.29 0.33 0.67 1.7
A2  0.05  0.07 -0.04  0.70  0.00 -0.07 0.52 0.48 1.1
A3 -0.03  0.03 -0.10  0.63  0.06  0.12 0.53 0.47 1.2
A4 -0.07  0.19 -0.08  0.41 -0.13  0.13 0.30 0.70 2.0
A5 -0.17  0.01 -0.19  0.46  0.11  0.22 0.48 0.52 2.3
C1  0.02  0.55  0.07 -0.04  0.19  0.09 0.36 0.64 1.3
C2  0.07  0.67  0.14  0.02  0.10  0.18 0.50 0.50 1.3
C3  0.01  0.56  0.06  0.07 -0.05  0.06 0.32 0.68 1.1
C4  0.05 -0.67  0.08 -0.05  0.04  0.29 0.59 0.41 1.4
C5  0.15 -0.55  0.18  0.00  0.10  0.04 0.43 0.57 1.5
E1 -0.14  0.10  0.60 -0.13 -0.08  0.09 0.40 0.60 1.4
E2  0.05 -0.03  0.68 -0.08 -0.07  0.02 0.56 0.44 1.1
E3  0.01  0.00 -0.33  0.15  0.39  0.24 0.48 0.52 3.0
E4 -0.05  0.03 -0.53  0.22  0.01  0.28 0.56 0.44 1.9
E5  0.15  0.26 -0.39  0.05  0.24  0.05 0.40 0.60 2.9
```

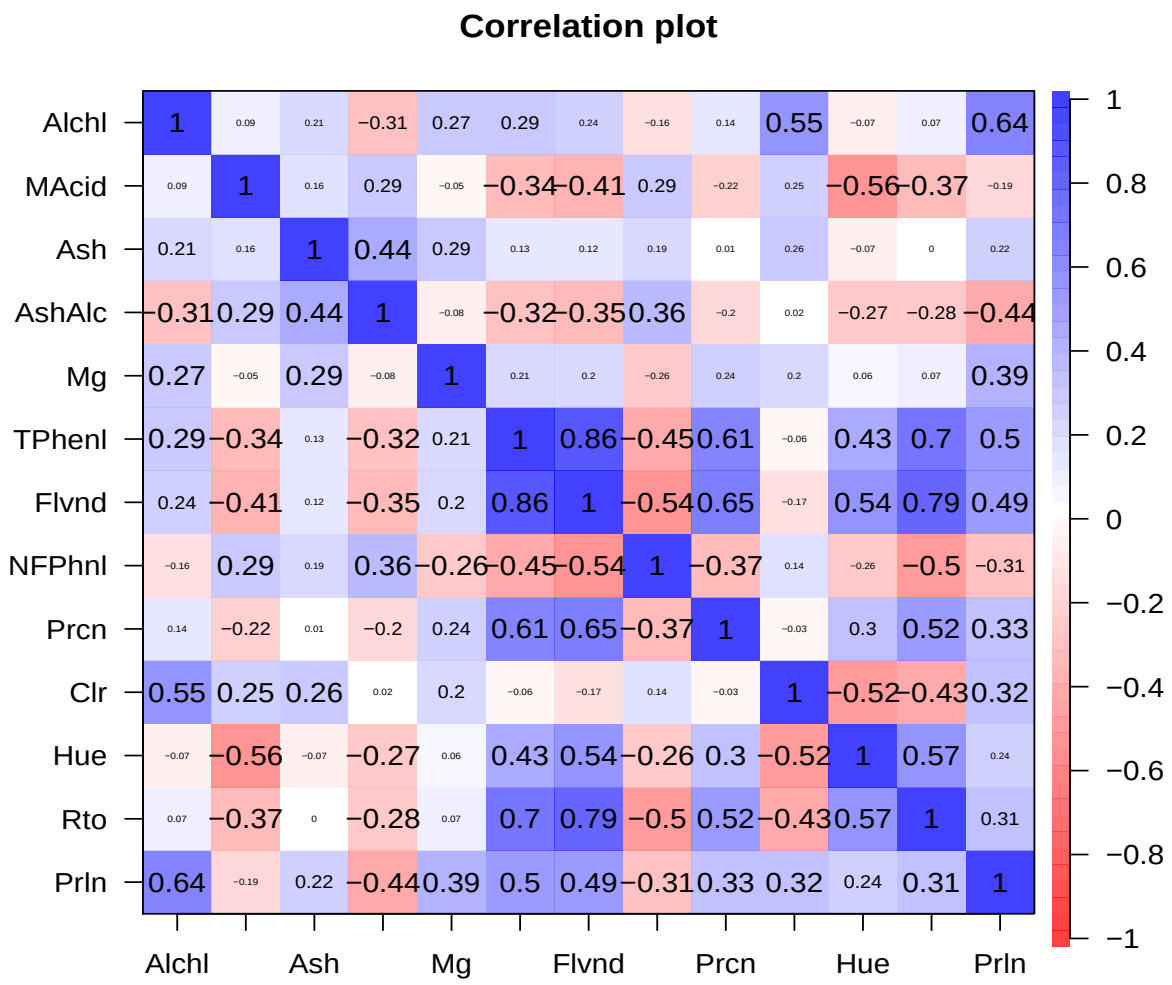
```
N1  0.85  0.00 -0.10 -0.07 -0.05  0.00 0.70 0.30 1.0
N2  0.83  0.02 -0.05 -0.04  0.00 -0.08 0.67 0.33 1.0
N3  0.67 -0.03  0.14  0.08  0.03  0.08 0.54 0.46 1.1
N4  0.43 -0.14  0.43  0.08  0.09  0.06 0.51 0.49 2.4
N5  0.44  0.00  0.24  0.17 -0.13  0.15 0.35 0.65 2.4
O1 -0.05  0.07 -0.03 -0.04  0.57  0.07 0.35 0.65 1.1
O2  0.12 -0.08 -0.02  0.09 -0.42  0.31 0.30 0.70 2.2
O3 -0.01  0.01 -0.10  0.04  0.66  0.04 0.49 0.51 1.1
O4  0.08 -0.03  0.35  0.15  0.36 -0.01 0.25 0.75 2.5
O5  0.04 -0.03 -0.05 -0.03 -0.49  0.35 0.36 0.64 1.9

                       MR2  MR3  MR1  MR5  MR4  MR6
SS loadings           2.50 2.11 2.18 1.96 1.74 0.76
Proportion Var        0.10 0.08 0.09 0.08 0.07 0.03
Cumulative Var        0.10 0.18 0.27 0.35 0.42 0.45
Proportion Explained  0.22 0.19 0.19 0.17 0.15 0.07
Cumulative Proportion 0.22 0.41 0.60 0.78 0.93 1.00

 With factor correlations of
      MR2   MR3   MR1   MR5   MR4   MR6
MR2  1.00 -0.19  0.25 -0.10  0.01  0.14
MR3 -0.19  1.00 -0.22  0.18  0.20  0.00
MR1  0.25 -0.22  1.00 -0.30 -0.19 -0.11
MR5 -0.10  0.18 -0.30  1.00  0.23  0.18
MR4  0.01  0.20 -0.19  0.23  1.00  0.04
MR6  0.14  0.00 -0.11  0.18  0.04  1.00

Mean item complexity =  1.7

Test of the hypothesis that 6 factors are sufficient.
The degrees of freedom for the null model are  300  and the objective function was
7.48 with Chi Square of  18146.07
The degrees of freedom for the model are 165  and the objective function was 0.38

The root mean square of the residuals (RMSR) is  0.02
The df corrected root mean square of the residuals is  0.03

The harmonic number of observations is  2436 with the empirical chi square 552.63
with prob <  2.5e-43
The total number of observations was  2436  with Likelihood Chi Square =  916.62
with prob <  1.6e-104

Tucker Lewis Index of factoring reliability =  0.923
RMSEA index =  0.043  and the 90 % confidence intervals are  0.041 0.046
BIC =  -370.06
Fit based upon off diagonal values = 0.99
```

```
Measures of factor score adequacy
                                                MR2  MR3  MR1  MR5  MR4  MR6
Correlation of (regression) scores with factors 0.93 0.89 0.89 0.88 0.86 0.76
Multiple R square of scores with factors        0.87 0.79 0.79 0.78 0.74 0.57
Minimum correlation of possible factor scores    0.73 0.59 0.59 0.55 0.48 0.15

> biplot.psych(fa(X,6),pch=".",choose=c(1,2,3))          # library(psych)
```

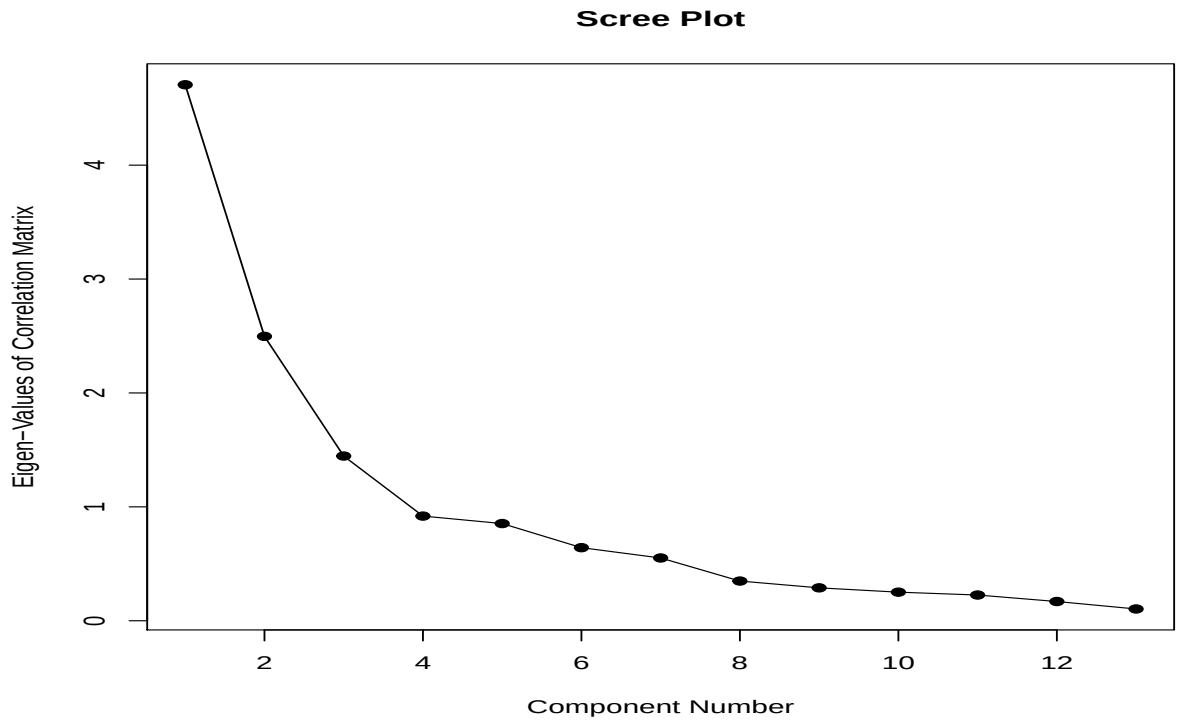


Example 2: The second data set is downloaded from the UC Irvine's Machine Learning data repository. The data set is comprised of observations on 13 quantitative variables resulting from chemical analysis of wines grown in the same region in Italy but derived from three different cultivars (cultivar labels are are also given *i.e.* it's a labeled data set). Very brief descriptions (essentially just the names) of these 13 variables may be found in `https://archive.ics.uci.edu/ml/datasets/Wine` (click/tap it!). While the actual task is one of supervised learning of being able to classify the three cultivars from these 13 variables, here we first undertake the (unsupervised learning task of) feasibility of reducing these (13) dimensions for basic interpretative purpose (which may or may not be useful for the subsequent classification task).

```
> d<-read.table("wine.data")
> X<-d[,2:14]
> names(X)<-c("Alchl","MAcid","Ash","AshAlc","Mg","TPhenl","Flvnd","NFPhnl",
+ "Prcn","Clr","Hue","Rto","Prln")

> cor.plot(X)     # library(psych)
```

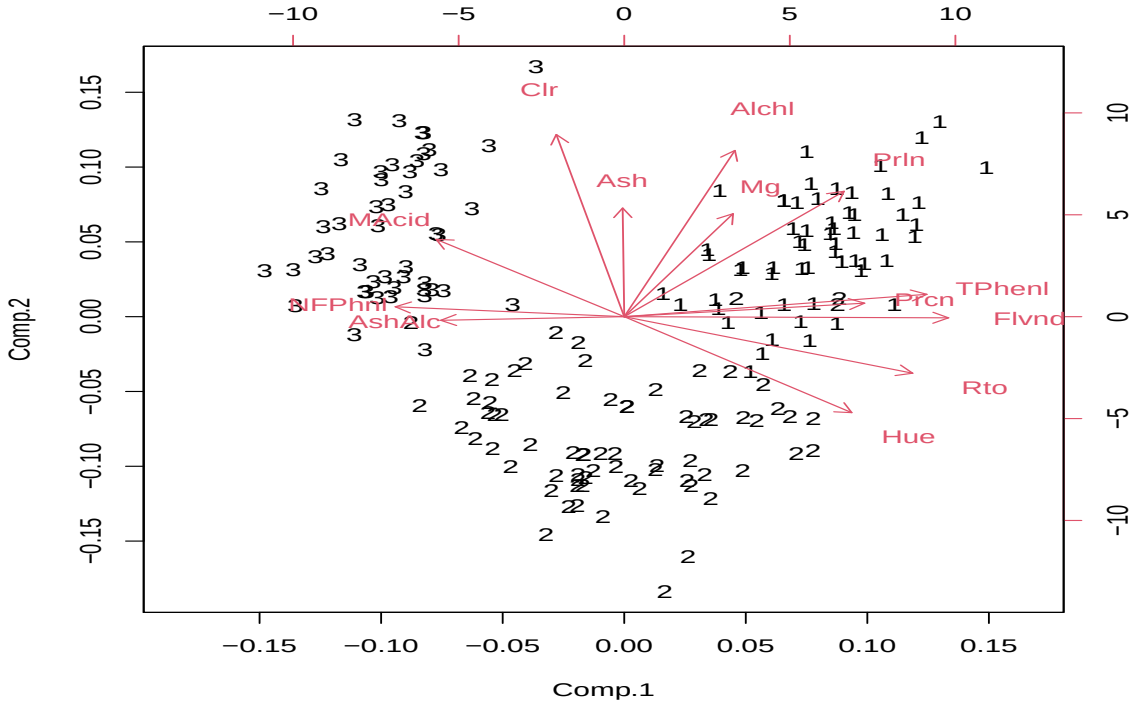



```
> pc.c<-princomp(X,cor=T)

> pc.c$sd^2
   Comp.1    Comp.2    Comp.3    Comp.4    Comp.5    Comp.6    Comp.7    Comp.8
4.7058503 2.4969737 1.4460720 0.9189739 0.8532282 0.6416570 0.5510283 0.3484974
   Comp.9   Comp.10   Comp.11   Comp.12   Comp.13
0.2888799 0.2509025 0.2257886 0.1687702 0.1033779

> plot(1:13,pc.c$sd^2,pch=19,xlab="Component Number",ylab="Eigen-Values of Correlation
Matrix",main="Scree Plot")
> lines(c(1:13),pc.c$sd^2)
```




```
> summary(pc.c)

Importance of components:
                          Comp.1    Comp.2    Comp.3    Comp.4     Comp.5
Standard deviation     2.1692972 1.5801816 1.2025273 0.9586313 0.92370351
Proportion of Variance 0.3619885 0.1920749 0.1112363 0.0706903 0.06563294
Cumulative Proportion  0.3619885 0.5540634 0.6652997 0.7359900 0.80162293

                           Comp.6     Comp.7     Comp.8     Comp.9    Comp.10
Standard deviation     0.80103498 0.74231281 0.59033665 0.53747553 0.50090167
Proportion of Variance 0.04935823 0.04238679 0.02680749 0.02222153 0.01930019
Cumulative Proportion  0.85098116 0.89336795 0.92017544 0.94239698 0.96169717
                          Comp.11    Comp.12     Comp.13
Standard deviation     0.47517222 0.41081655 0.321524394
Proportion of Variance 0.01736836 0.01298233 0.007952149
Cumulative Proportion  0.97906553 0.99204785 1.000000000

> pc.c$loadings

Loadings:
       Comp.1 Comp.2 Comp.3 Comp.4 Comp.5 Comp.6 Comp.7 Comp.8 Comp.9 Comp.10
Alchl   0.144  0.484  0.207         0.266  0.214         0.396  0.509  0.212
MAcid  -0.245  0.225        -0.537         0.537 -0.421                -0.309
Ash            0.316 -0.626  0.214  0.143  0.154  0.149 -0.170 -0.308
AshAlc -0.239        -0.612                -0.101  0.287  0.428  0.200
```

```
Mg      0.142  0.300 -0.131  0.352 -0.727        -0.323 -0.156  0.271
TPhenl  0.395        -0.146 -0.198  0.149                -0.406  0.286 -0.320
Flvnd   0.423        -0.151 -0.152  0.109                -0.187        -0.163
NFPhnl -0.299        -0.170  0.203  0.501 -0.259 -0.595 -0.233  0.196  0.216
Prcn    0.313        -0.149 -0.399 -0.137 -0.534 -0.372  0.368 -0.209  0.134
Clr             0.530  0.137                -0.419  0.228                -0.291
Hue     0.297 -0.279         0.428  0.174  0.106 -0.232  0.437        -0.522
Rto     0.376 -0.164 -0.166 -0.184  0.101  0.266                0.137  0.524
Prln    0.287  0.365  0.127  0.232  0.158  0.120         0.120 -0.576  0.162

       Comp.11 Comp.12 Comp.13
Alchl   0.226   0.266
MAcid           -0.122
Ash     0.499          -0.141
AshAlc -0.479
Mg
TPhenl -0.304   0.304  -0.464
Flvnd                   0.832
NFPhnl -0.117           0.114
Prcn    0.237          -0.117
Clr             -0.604
Hue             -0.259
Rto             -0.601  -0.157
Prln   -0.539

               Comp.1 Comp.2 Comp.3 Comp.4 Comp.5 Comp.6 Comp.7 Comp.8 Comp.9
SS loadings     1.000  1.000  1.000  1.000  1.000  1.000  1.000  1.000  1.000
Proportion Var  0.077  0.077  0.077  0.077  0.077  0.077  0.077  0.077  0.077
Cumulative Var  0.077  0.154  0.231  0.308  0.385  0.462  0.538  0.615  0.692

               Comp.10 Comp.11 Comp.12 Comp.13
SS loadings      1.000   1.000   1.000   1.000
Proportion Var   0.077   0.077   0.077   0.077
Cumulative Var   0.769   0.846   0.923   1.000

> cultivar<-c(rep("1",59),rep("2",71),rep("3",48))
> biplot(pc.c,xlabs=cultivar)
```

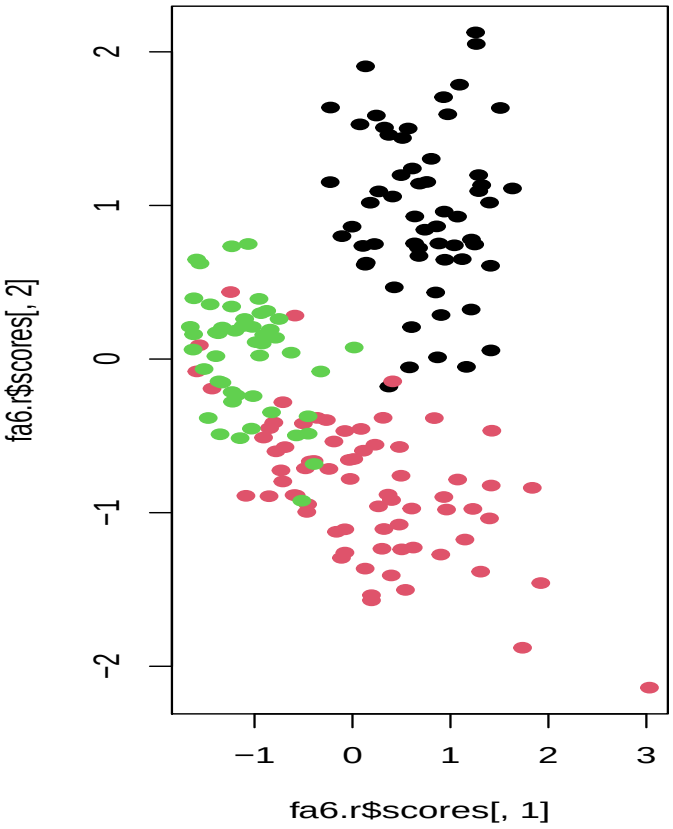


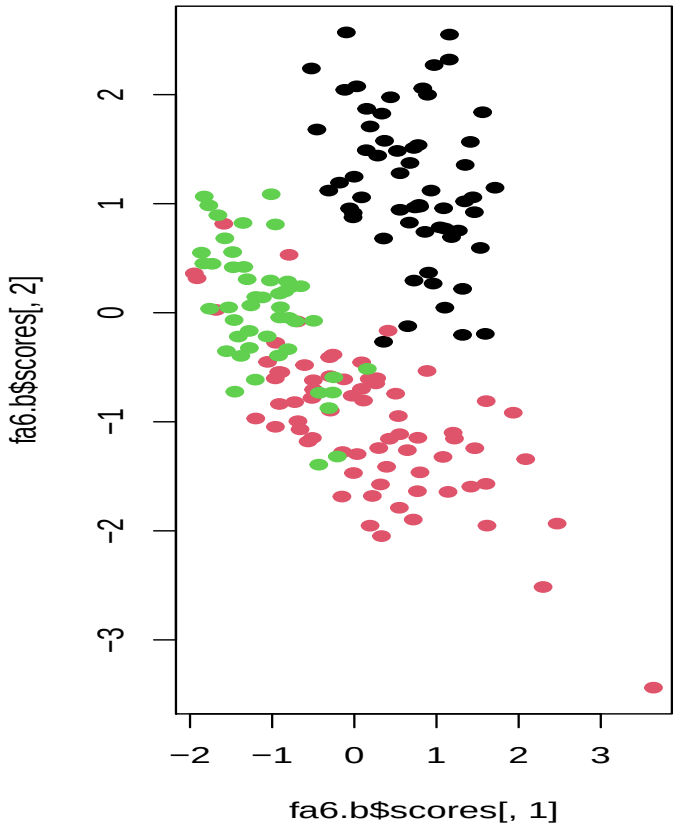

```
> factanal(X,2)$PVAL
1.485595e-32
> factanal(X,3)$PVAL
1.959095e-15
> factanal(X,4)$PVAL
1.444642e-05
> factanal(X,5)$PVAL
0.02056416
> factanal(X,6)$PVAL
0.3093393

> factanal(X,6)

Call:
factanal(x = X, factors = 6)

Uniquenesses:
 Alchl  MAcid    Ash AshAlc     Mg TPhenl  Flvnd NFPhnl   Prcn    Clr    Hue
 0.308  0.285  0.005  0.393  0.005  0.198  0.055  0.583  0.482  0.005  0.349
   Rto   Prln
 0.247  0.239

Loadings:
       Factor1 Factor2 Factor3 Factor4 Factor5 Factor6
Alchl    0.142   0.763           0.254   0.124
MAcid  -0.245           0.129           0.793
```

```
Ash             0.177   0.967                   0.138
AshAlc -0.233  -0.463   0.541           0.193
Mg      0.148   0.196   0.114                   0.958
TPhenl  0.841   0.252                  -0.165
Flvnd   0.907   0.221          -0.131  -0.233
NFPhnl -0.516  -0.143   0.261   0.108   0.114  -0.194
Prcn    0.697                                  0.129
Clr    -0.133   0.410   0.126   0.875   0.149
Hue     0.379                  -0.475  -0.524
Rto     0.750                  -0.385  -0.177
Prln    0.322   0.769                  -0.155   0.191

               Factor1 Factor2 Factor3 Factor4 Factor5 Factor6
SS loadings      3.268   1.775   1.355   1.265   1.138   1.046
Proportion Var   0.251   0.137   0.104   0.097   0.088   0.080
Cumulative Var   0.251   0.388   0.492   0.589   0.677   0.757

Test of the hypothesis that 6 factors are sufficient.
The chi square statistic is 17.16 on 15 degrees of freedom.


> factanal(X,5)

Call:
factanal(x = X, factors = 5)

Uniquenesses:
 Alchl  MAcid    Ash AshAlc     Mg TPhenl  Flvnd NFPhnl   Prcn    Clr    Hue
 0.348  0.107  0.005  0.399  0.792  0.198  0.055  0.623  0.512  0.005  0.384
   Rto   Prln
 0.252  0.204

Loadings:
       Factor1 Factor2 Factor3 Factor4 Factor5
Alchl   0.138   0.737           0.264   0.125
MAcid  -0.247           0.131   0.132   0.893
Ash             0.352   0.931
AshAlc -0.246  -0.367   0.613           0.146
Mg      0.112   0.412   0.150
TPhenl  0.842   0.270                  -0.133
Flvnd   0.912   0.238          -0.146  -0.186
NFPhnl -0.505  -0.153   0.266   0.113   0.122
Prcn    0.685   0.102
Clr    -0.147   0.448           0.869   0.103
Hue     0.405                  -0.508  -0.428
Rto     0.756                  -0.388  -0.137
Prln    0.310   0.826                  -0.109
```

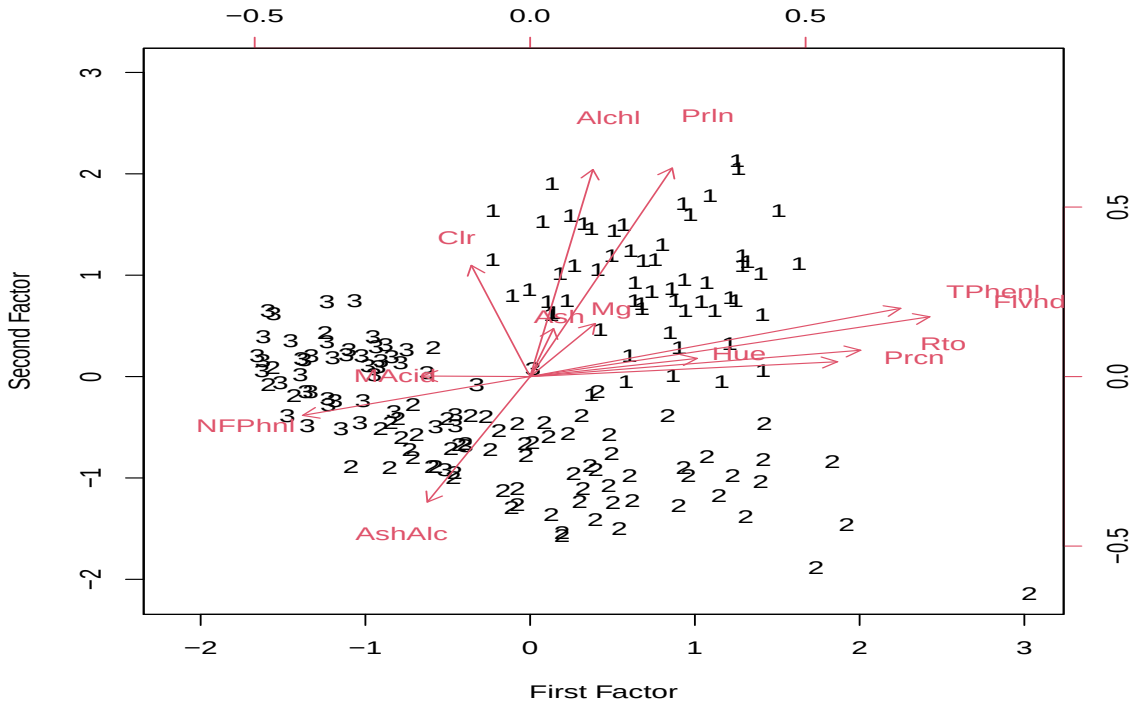
```
                Factor1 Factor2 Factor3 Factor4 Factor5
SS loadings       3.274   2.030   1.381   1.298   1.135
Proportion Var    0.252   0.156   0.106   0.100   0.087
Cumulative Var    0.252   0.408   0.514   0.614   0.701

Test of the hypothesis that 5 factors are sufficient.
The chi square statistic is 38.86 on 23 degrees of freedom.
The p-value is 0.0206

> factanal(X,4)

Call:
factanal(x = X, factors = 4)

Uniquenesses:
 Alchl  MAcid    Ash AshAlc     Mg TPhenl  Flvnd NFPhnl   Prcn    Clr    Hue
 0.372  0.703  0.005  0.402  0.790  0.195  0.056  0.625  0.511  0.163  0.378
   Rto   Prln
 0.255  0.194

Loadings:
       Factor1 Factor2 Factor3 Factor4
Alchl   0.143   0.743  -0.224
MAcid  -0.302          -0.409   0.189
Ash             0.355           0.931
AshAlc -0.227  -0.359  -0.208   0.612
Mg      0.113   0.420           0.144
TPhenl  0.833   0.281   0.179
Flvnd   0.891   0.242   0.304
NFPhnl -0.494  -0.146  -0.199   0.264
Prcn    0.685   0.122
Clr             0.529  -0.740
Hue     0.355           0.698
Rto     0.708           0.492
Prln    0.288   0.836   0.128


                Factor1 Factor2 Factor3 Factor4
SS loadings       3.095   2.143   1.722   1.391
Proportion Var    0.238   0.165   0.132   0.107
Cumulative Var    0.238   0.403   0.535   0.642

Test of the hypothesis that 4 factors are sufficient.
The chi square statistic is 76.92 on 32 degrees of freedom.
The p-value is 1.44e-05
```

```
> fa6.r<-factanal(X,6,scores="regression")
> fa6.b<-factanal(X,6,scores="Bartlett")

> plot(fa6.r$scores[,1],fa6.r$scores[,2],col=cultivar,pch=19)
> plot(fa6.b$scores[,1],fa6.b$scores[,2],col=cultivar,pch=19)
```






```
> biplot(matrix(c(fa6.r$scores[,1],fa6.r$scores[,2]),ncol=2),
+ matrix(c(fa6.r$loadings[,1],fa6.r$loadings[,2]),ncol=2),
+ xlabs=cultivar,ylabs=names(X),xlab="First Factor",ylab="Second Factor")
```




```
> biplot(matrix(c(fa6.r$scores[,3],fa6.r$scores[,4]),ncol=2),
+ matrix(c(fa6.r$loadings[,3],fa6.r$loadings[,4]),ncol=2),
+ xlabs=cultivar,ylabs=names(X),xlab="Third Factor",ylab="Fourth Factor")
```