

Probability Models Problems

1. A professor suddenly gave a pop-quiz (a test which is not announced in advance) containing 10 multiple choice questions having 4 choices, with only one of them being correct, in one of his Statistics classes having 25 students. Assume that every student in the class answers independently of one another, but being completely unprepared for the quiz, randomly guesses the answers independently for all the questions. Answer the following:

- a. What is the probability of a student getting at least half the answers correct in the quiz? How many correct answers would you *expect* a student to get? Interpret this value. What is the most likely number of correct answers that a student will get in the pop-quiz?
- b. The professor decides to assign the grades of A, B, C, D and F in the pop-quiz, if the number of correct answers are 9-10, 6-8, 3-5, 1-2 and 0 respectively. Find the probability of the class getting 0 As, 2 Bs, 10 Cs, 10 Ds and 3 Fs in the quiz. Find the correlation coefficient between the number of Cs and number of Ds and interpret its value. Find the median of “No. of Fs” and interpret its value.

2. The number of typing mistakes made by a certain typist is found to have a Poisson distribution with a mean of 0.25 mistakes per page. The typist is asked to prepare a three-page document each for 18 different clients. Find the probability of at least n clients receiving typo-free documents for $n=2, 5$ and 10. Find the most likely number of clients receiving documents containing at most one typo, and its IQR.

3. The daily sales of TV sets made by a certain store has a Poisson distribution with a mean of 1.2. The store is open for 6 days a week and the sale of one day is independent of another. Answer the following:

- a. What is the distribution of weekly sales of TV sets made by the store?
- b. What is the most likely number of TV sets that will be sold by the store in a given week?
- c. In how many weeks in a given year, which has 52 weeks, can one expect the weekly sales to exceed 10 TV sets? In how many weeks in the year is it most likely to have its sale exceeding 8 TV sets?
- d. For logistic reasons the store has to maintain a weekly inventory. At least how many TV sets should the store have in stock every week, so that one can be 95% certain of meeting the weekly demand?

4. Daily demand for a popular book in a bookstore, which remains open on 5.5 days a week, has a Poisson distribution with $\lambda = 4$. How many weeks is one expected to wait before the demand exceeds 25 copies in a week? A run is defined as the collection of weeks between the two registering a sale exceeding 25. The run length is the number of weeks in it. Find the variance of the run length.

5. The “true” (some people tend to lie in their resumé) number of overseas projects completed by software professionals, appearing in the interview for Project Leader positions in Company Z, has a Poisson distribution with mean 2.5. Only candidates who have completed at least 4 overseas projects will be considered for the positions. Answer the following:

- a. What is the probability that among the first 5 candidates interviewed, there will be at least one candidate worth consideration?

- b. How many candidates are expected to be interviewed to get two candidates worth consideration?
 - c. The probability of selecting a considered candidate (with the requisite experience of having completed at least 4 overseas projects) for the Project Leader position is 0.8. What can one say with 95% certainty about the maximum number of candidates that need to be interviewed for selecting 5 Project Leaders?
6. In a shipment of 1000 mangoes there are 100 rotten ones. A random sample of 50 mangoes are taken without replacement. You may find Stirling's approximation $n! \approx \sqrt{2\pi n} n^{n+\frac{1}{2}} e^{-n}$ (for large n) useful in answering the following:
- a. Find the probability of finding at most 5 rotten mangoes in the sample.
 - b. What is the probability of finding exactly 8 rotten ones?
 - c. What is the most likely number of rotten mangoes in the sample?
7. In a lot of 1000 screws there are 10 which are defective (not properly grooved). A lot is accepted, if in a random sample of 50 drawn without replacement, at most one is defective. Find the probabilities of this particular lot, as well as at least 20 out of 25 such lots (of size 1000 with 10 defective) getting accepted.
8. It is believed that at most 10% of the people approached in a vacation spot, will be interested in a time-share-holiday of ABC Resort. What can be said about the probability of having to approach at least 8 people before someone shows interest in the time-share-holiday of ABC Resort? At least how many people one is expected to approach to get someone showing interest in the time-share-holiday of ABC Resort?
- 9 a. Show that if X_1 and X_2 are independent Poisson random variables, the conditional distribution of X_1 given $X_1 + X_2$ is Binomial.
- b. The daily sales of refrigerators made by two retail outlets A and B owned by Ms.X have independent Poisson distributions with respective means of 3.2 and 4.1. On a certain day if A and B together sales 10 refrigerators, what is the most likely number of refrigerators that are sold by A?
10. Monthly demand for branded soap in a medium sized village is estimated to have a Poisson distribution with a mean of 36 packs. However the business is profitable (essentially for the cost of distribution) only if the demand exceeds 30 packs per month in a village. Answer the following:
- a. What is the probability that at least 75% of the villages will turn out to be profitable in a pilot marketing trial of 200 villages?
 - b. What is the most likely number of villages (among these 200) where the business will be profitable?
11. In an operation theater the main light-bulb is replaced after every 1000 hours of its burning if it has not failed already in which case it is immediately replaced. The life of the light-bulb has an exponential distribution with a mean of 800 hours. Find the *expected* number of hours the light-bulb burns.

12. Customers arrive at a counter at a homogeneous Poisson rate of 0.4 per minute. A discount coupon is issued to a customer if s/he arrives more than 5 minutes after the arrival of the last customer. What is the probability that there will be more than 4 customers between two coupon receiving customers? How many customers would you expect to see not receiving a coupon between two coupon receiving customers?

13. A chocolate manufacturer mixes dry fruits at the proportion of 0.1 per gram in its mix. A girl purchases 10 such chocolates of 100 grams each. What is the probability of finding at least 2 chocolates each containing more than 12 pieces of dry fruits? What is the most likely number of chocolates (among the 10 purchased by the girl) that contain more than 12 pieces of dry fruits?

14. The distance traveled by a ball hit by a certain golfer with a certain club has a normal distribution with a mean of 250 metres and a standard deviation of 25 metres. For practice, if the golfer hits a ball in a direction which has two impediments, each one 50 metre long, at distances 195 and 285 metres, 100 times, then what is the probability of the golfer staying clear of the impediments at least 60 times?

15. The amount of time it takes to properly interview a candidate for a certain kind of job, has a normal distribution with a mean of 20 minutes and a standard deviation of 5 minutes. Assuming that all candidates show up for the interview, at most how many candidates should be called in a day with 8 working hours, so that one can be more than 99% certain of finishing the interviews within the working day?

16. The lifetime of an IC chip has an exponential distribution with a mean of 5 years, and the cost of manufacturing a single chip is Rs.10. A batch of 10,000 chips are to be sold with a replacement warranty of 1 year. The cost of replacing a failed chip within the warranty period of 1 year is its manufacturing cost of Rs.10 plus a loss in customer satisfaction elicited at Rs.2 per chip. At what price should a chip be sold to ensure a profit of Rs.10,000 with 95% assurance from the manufactured batch of 10,000 chips? Ignore the possibility of multiple replacements of a chip within the warranty period.

17. 20% of people refuse to speak to a tele-marketing executive offering personal loan from a bank, and among those who do, the amount of time they agree to spend for listening to the benefits, terms and conditions of the loan has an exponential distribution with a mean of 5 minutes. If a person spends more than 10 minutes with the executive, the probability of applying for a personal loan is 0.6 and otherwise the probability is only 0.1. At least how many persons does the tele-marketing executive need to call before being able to receive at least one loan application with a minimum 99% certainty?

18. Time taken to receive payment for an order has an exponential distribution with a mean of 10 working days after dispatching the finished goods. On a certain day 10 shipments of finished goods were dispatched with receivable amounts of Rs.10,000 each for 6 of them and Rs.15,000 each for 4 of them. The payable account of Rs.100,000 for the raw materials purchased for manufacturing these 10 shipments of finished goods needs to be settled within 7 working days. This payable account of Rs.100,000 will be settled from the amount raised from these 10 receivable accounts and the rest by borrowing money from the parent financing institution. Answer the following.

a. What is the probability that money needs to be borrowed from the parent financing institution for settling the payable account?

- b. What is the amount of money that is *expected* to be borrowed from the parent financing institution?

19. A project leader in an application software company has 3 programmers in her team. She usually, a) breaks a project into a number of modules of approximately similar sizes and difficulty levels, b) explains the modules in a meeting with her team, c) instructs each member of her team to write the code for each module and submit it to her as soon as the code is ready, d) uses the first submitted code for a module, and e) informs all the members of her team to abandon working on the current module and proceed with the next one as soon as she receives the code of a completed module. She has observed that the amount of time the programmers take to write the code for a module are independent of each other and has exponential distributions with respective means of 1, 1.1, and 1.2 hours.

- a. What is the distribution of the amount of time she has to wait for the code of a module to be ready?
b. How much time is she expected to wait for the code of a module to be ready?

She has broken up project Z into 12 modules and used her style of management to get the codes written for the modules. Assume that the time taken for the code of each module of project Z to reach her are independent of each other.

- c. What is the probability that she will get at least 10 modules of project Z, codes of each one of which got written within half an hour?
d. What is the distribution of the amount of time she has to wait for the codes of all the 12 modules of project Z to be ready?
e. What is the probability that she has to wait for more than 8 hours (a full working day) for the codes of all the 12 modules of project Z to be ready?
f. She estimates that she will need an extra hour to integrate the codes of the 12 modules to finish project Z. What amount of time should she ask from her client for finishing project Z so that she is 95% sure of completing it by then?

20. Newly manufactured electrical cables are subject to a stress-strength test before marketing. A random amount of electrical stress is applied to each cable and it survives if its strength exceeds the applied stress. Suppose the applied stress has an exponential distribution with mean 100, and the strength distribution of the cables is independent of the applied stress and has an exponential distribution with mean 200. If 1000 such cables are tested on a given day, what is the probability that at least 700 survive the stress-strength test?

21. The amount of time (in minutes) required to finish a job by a machine has an exponential distribution with a mean of 4 minutes. If 50 such jobs are submitted to the machine, will it be safe to assume that no more than half of them will take more than 5 minutes to be finished?

22. The diameter of the ball bearing balls produced by a manufacturing process has a Normal distribution with a mean of 5mm and a standard deviation of 0.05mm. The lower and upper specification limits(for the diameter) set by one of the customers are 4.9mm and 5.1mm respectively. If 1000 such balls are shipped to that customer what is the probability that the diameters of at least 950 of the balls fall within the specification limits? The

diameters of how many balls(among those 1000 shipped) are most likely to be within the specification limits?

23. The quality control inspector of a bicycle company takes a random sample of 50 ball-bearing balls from incoming lots containing 1000 balls without replacement, and rejects the lot if more than 1 of the sampled balls has a diameter outside the specification limits of 4.99 mm and 5.01 mm. If the diameter of the incoming balls has a Normal distribution with a mean of 5.001 mm and a standard deviation of 0.0055 mm, what is the probability of a lot getting rejected?

24. The daily amount of milk produced by a certain dairy has a Normal distribution with a mean of 150,000 litres. The daily domestic demand from the dairy is independent of the milk produced by the dairy and has a Normal distribution with a mean of 100,000 litres and a standard deviation of 5,000 litres. Government stipulates the dairy to meet the domestic demand first and the excess milk produced by the dairy is exported with a profit of \$50 per 1000 litres. The dairy has reached a saturation point where it cannot increase its mean daily production any further.

- a. If the dairy wants to make a daily profit of at least \$2000 with a minimum of 95% certainty from its export, what is the maximum amount of variability in its daily production it can afford?
- b. However much it reduces the variability in its production process, why cannot the dairy fix a target of more than \$2500 daily profit from its export with more than 50% certainty?
- c. What maximum target of daily profit from export is possible for the dairy with a minimum of 95% certainty?

25. The price of a dinner in a restaurant in a city has a Normal distribution with a mean of Rs.150. If 85.85% of the bills exceed Rs.124.25, what is the standard deviation of the price?

26. Daily amount of claims an insurance company has to pay is Normally distributed with a mean of Rs.1 lakh and a standard deviation of Rs.10,000. The amount of premium it collects per policy averages out to have a daily mean of Rs.50 and a standard deviation of Rs.10. At least how many policies should be live on a daily basis, so that the insurance company can cover the insurance claim amounts from at most 50% of its revenues from the policy premiums with a probability of at least 0.95?

27. Trading takes place in a stock exchange like BSE by matching limit orders *i.e.* buyers and sellers place orders with their respective bid and ask prices, and subsequently a transaction takes place if a buyer's bid price exceeds the ask price of a seller. A buy-order with a bid price of Rs.125 for a certain scrip is place at a time when there are no outstanding sell orders. Suppose the sell orders arrive at a homogeneous Poisson rate of 1 every 5 minutes with an ask price that is Normally distributed with a mean of Rs.150 and a standard deviation of Rs.20. Also assume that no buy-order comes in between. Answer the following.

- a. What is the probability that the buyer has to wait at least an hour to buy the scrip?
- b. Find the mean and variance of the amount of time the buyer has to wait for buying the scrip.

28. Car and Housing loan applications arrive at a bank during its customer interaction hours of 10AM to 2PM at independent homogeneous Poisson rates of 2.5 and 1 per hour respectively. Furthermore the requested amounts of Car and Housing loans are independently distributed with respective Expectations of Rs.2 lakhs and Rs.8 lakhs with Standard Deviations Rs.25,000 and Rs.1 lakh respectively. Answer the following:

- a. What is the probability of receiving at least 10 loan applications (Car and Housing taken together) on any given working day? parent financing institution for settling the payable account?
- b. If on a given working day there are 18 loan applications, what is the probability of at least 10 of them are for Cars?
- c. What is the Expectation and Standard Deviation of the total amount of loan requests on any given working day?

29. Customers arrive at the check-out counter of a super-market at a homogeneous Poisson rate of 1 per minute. Answer the following:

- a. What is the probability of the 25th customer arriving after half an hour of opening the check-out counter?
- b. Given that the third customer arrived at the 5th minute, what is the correlation between the arrival times of the first and the second customer?

30. During the Saturday evening peak hours, customers arrive at the check-out counter of a departmental store at a Poisson rate of 6 per minute. The service time for each customer has an exponential distribution with a mean of 0.5 minutes, which is assumed to be independent of the inter-arrival times of the customers.

- a. What is the probability that a customer has to wait to be served?
- b. How many check-out counters should be opened during the Saturday evening peak hours so that the probability that a customer has to wait is 0.2, assuming that only a single queue is maintained for all the counters and the service times of the different check-out counters are mutually independent of each other?