MG 221: One Sample Problems

1. Let $X_1, X_2, \ldots, X_n$ be a random sample from a Poisson(λ) population (the p.m.f. of a Poisson(λ) population is given by $p(x|\lambda) = e^{-\lambda}\frac{\lambda^x}{x!}$ for $x = 0, 1, 2, \ldots$). Answer the following:

a. Find the UMVUE of λ and its SE.

b. Find the MLE of λ and its asymptotic SE.

c. Find the UMP test for testing $H_0 : \lambda = \lambda_0$ versus $H_a : \lambda > \lambda_0$.

d. Find the LR test for testing $H_0 : \lambda = \lambda_0$ versus $H_a : \lambda \neq \lambda_0$.

e. Given that 5, 2, 1, 5, 4, 0, 3, 3, 1, 2 are the number of daily chain-santching incidents in the city (which may be assumed to follow a Poisson distribution) on 10 randomly selected days, give the p-value of the pair of hypothesis given in **c** above, for $\lambda_0 = 2$. Also give the UMVUE and its SE as well as the MLE and its asymptotic SE of λ based on this numerical sample.

2. Water levels at a certain point in a river on 30 randomly selected days in a given year, compared to its normal level, in sorted order are as follows:

-20.8	-14.6	-14.1	-10.0	-9.0	-8.0	-7.1	-6.6	-5.1	-4.9	-4.6	-3.5	-2.7	-1.9	-1.9
0.2	0.6	1.1	1.8	2.5	3.3	4.0	4.4	4.6	5.2	5.4	5.8	6.1	7.0	9.7

A negative means it was below normal, with the number indicating the number of cm below normal. Similarly a positive means above normal and the number indicates the number of cm above normal. Answer the following:

a. The river is said to be going through a drought that year if the average level remains below normal. Is there sufficient evidence to indicate that the river is going through a draught that year?

b. Assuming that the deviance from normal level to be Normally distributed, give a 95% confidence interval for the variance of the deviance.

c. Is there sufficient evidence to indicate that the fall from the normal level was more than 2 cm for at least 25% of the days?

3. A project-leader believes that more than 60% of the people feel motivated to work if she works with them on a project. In the recent past she has worked with 15 people on different projects, she is leading, and has found 10 of them to be motivated. Is there strong evidence justifying her belief?

4. Suppose you want to estimate the proportion of IISc students who own a two-wheeler, with a 90% confidence interval and an error tolerance of at most $\pm$ 0.1. At least how large a sample should you draw?

5. An economist wants to estimate the proportion of house-holds living in a district without electricity, with 95% confidence with an error bound of at most ± 0.03. At least how many house-holds needs to be sampled?

6. A broker has correctly predicted the behavior of 7 stocks (whether the value will go up or go down) in a portfolio of 10 stocks. Is there sufficient evidence to suggest that the broker indeed has some mechanism better than guessing in predicting the stock price behavior (in terms of whether the value goes up or down)?

7. A sociologist is interested in estimating the proportion of *adivasis* who have recently changed their religion in a certain district in Tamilnadu. At least how many *adivasis* should she include in her sample, so that she can estimate that proportion with 95% confidence with a maximum error margin of ± 0.05?

8. Annual export amounts (in thousands of US \$'s) of 12 randomly chosen small-scale software companies in the last financial year is as follows::

11,	12,	8,	4,	12,	7,	14,	10,	6,	11,	9,	10

Assume that the annual export figure of the small-scale software industry has a Normal distribution. Answer the following:

a. Find a 95% confidence interval for the mean annual export figure of the small-scale software industry.

b. Give an upper bound for the variance of the annual export figure of the small-scale software industry, so that you are 90% confident.

9. Actual content (in ml) of tomato sauces in 10 randomly selected 250ml bottles in a bottling plant is as follows:

248,	249.5,	250.5,	247.5,	251,	250,	250.5,	249.5,	250,	249

Answer the following:

a. Find a 95% confidence interval for the mean content.

b. Is it safe to assume that the mean content of the bottles are maintained at 250ml?

c. The bottling process is said to be "out of control" if the variance of the contents exceed 1ml^2. Is there strong evidence suggesting that the bottling process has gone "out of control"?

d. Give an upper-bound for the variance, so that you are 99% confident.

10. The national average of yearly number of patents filed by scientists working in the research labs of the pharmaceuticals industry is 3.5. The average annual number of patents filed by 15 scientists working in Lab X are as follows:

4.8,	2.2,	1.4,	2.6,	5.2,	4.8,	3.9,	3.2,	1.7,	5.8,	1.9,	4.4,	3.7,	2.7,	3.5

Answer the following:

a. Is there any evidence suggesting the number of patents filed by the scientists working in Lab X is different from the national average?

b. Give a 95% confidence interval for the median of the average annual number of patents filed by scientists working in Lab X.

11. Packets containing 250 gms. of cereals are to be distributed to citizens in a flood-stricken region. Since the cereals are packeted in a haste by means of a measuring cup instead of proper weighing, the contents in the packets have a wide range of variability. Inspectors are instructed to ensure that 90% of the packets must contain at least 250 gms. of cereals. A sample of 15 packets of cereals chosen at random from a lot ready to be distributed have the following weights (in gms.):

272	320	218	270	280	245	255	260	233	262	252	290	255	260	248

Is it safe to conclude that 90% of the packets contain at least 250 gms. of cereals?

12. The CEO of a large organization is considering a mass scale promotion strategy for middle level managers to bring in young and fresh blood in their positions. However the CEO will take such a decision only if at least 75% of the middle level managers are at least 45 years old. A random sample of 20 middle level managers reveals the following ages:

48, 42, 54, 46, 52, 48, 49, 38, 47, 58,
49, 44, 47, 50, 46, 51, 46, 41, 47, 53

What should the CEO do?

13. The standard deviation of the amount of oil filled in 1 litre poly-packs is estimated to be 2 ml. from a random sample of 21 packets. The filling process is said to be under control if the population standard deviation is less than or equal to 1.5 ml. Answer the following:

a. Is there sufficient evidence to conclude that the filling process has gone out of control?

b. What can you say about the minimum value of the standard deviation of the filling process with 95% confidence?

14. The number of hours it takes to assemble the components of the engine compartment in newly manufactured automobiles in a sample of size 50 are as follows:

7.7	8.7	8.9	9.0	9.0	9.2	9.3	9.4	9.7	9.7	9.9	10.0	10.1
10.2	10.4	10.4	10.7	10.8	11.0	11.2	11.6	11.6	11.7	11.9	12.2	12.4
12.4	12.4	12.4	12.6	12.6	12.7	12.7	12.8	12.8	12.9	12.9	12.9	13.1
13.3	13.4	13.5	14.0	14.7	14.7	14.8	15.2	15.9	16.2	17.3		

Atleast how many hours need to be budgeted (for a subsequent sheduling problem) with 95% confidence so taht the assembly of engine compartment components is complete for 90% of the cars?

15. The number of withdrawals on Saturday nights from a particular ATM, which was observed for 100 such occasions, have the following frequency distribution:

No. of Withdrawals	7	8	9	10	11	12	13	14	15
Frequeucy	2	1	4	4	4	8	9	13	8
No. of Withdrawals	16	17	18	19	20	21	22	24	26
Frequeucy	8	7	11	6	5	5	1	3	1

Provide an upper bound such that you are 95% confident that 90% of the time the number of withdrawals on Saturday nights from that ATM would be less than that.