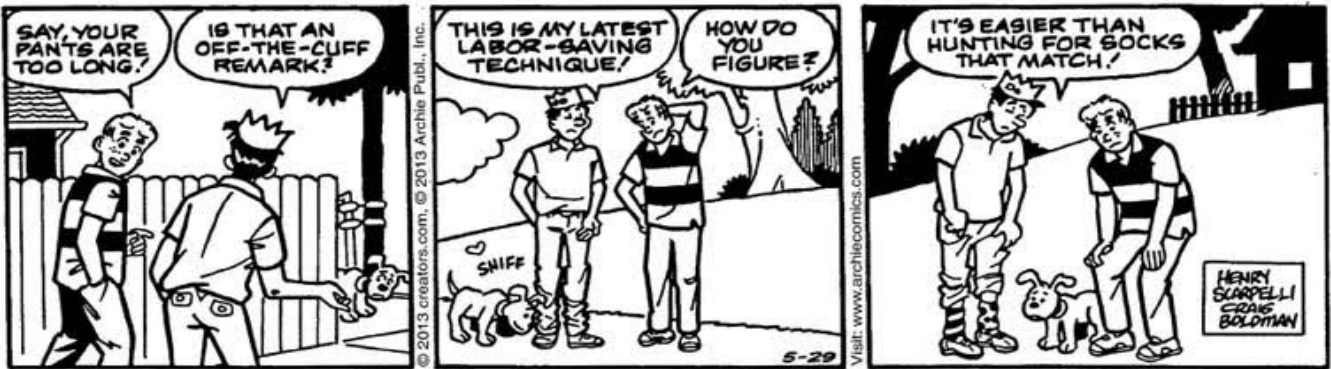


MG221: Applied Probability & Statistics

Midterm 2018

1.



Archie's friend Jughead above has a pair of yellow, two pairs of red, three pairs of green and four pairs of blue socks, from which he randomly picks up two socks to wear. For each pair, the one meant for the left foot is distinct from the right, but a left-right (or left-left or right-right) pair of same colour is indistinguishable from another same combo pair of the same colour. What are the probabilities of Jughead picking up two socks,

- a. comprising a "proper pair" (a left-right combo of the same color)? [5]
- b. of different colours? [5]
- c. of same color given that they are for the same foot? [5]
- d. for the same foot given that they are of different colours? [5]

2. A point is chosen at random from the interior of a right-angled, isosceles triangle with two perpendicular sides of unit length. Let X denote the distance of the chosen point from the hypotenuse. Answer the following:

- a. Find the median of X and interpret its value. [5+2=7]
- b. Find the mode of X and interpret its value. [4+2=6]
- c. Find the mean of X and interpret its value. [5+2=7]

3. The amount of time it takes a Customer Care Representative (CCR) to take care of a customer request has an exponential distribution with a mean of 5 minutes. These random times are further assumed to be independent from one customer to the next, and also from one CCR to another. Answer the following:

- a. Given that a CCR has already spent more than 7 minutes on a customer request, what is the probability that she will take at least another 3 minutes to take care of that request? [2]
- b. What is the probability that among 10 customers there are at least 2, whose requests took more than 10 minutes to take care of? [4]
- c. What is the probability of the total time to take care of the requests of 10 customers by a CCR exceeding one hour? [4]
- d. Prove that for large λ , $\text{Poisson}(\lambda) \sim \text{AN}(\lambda, \lambda)$. [5]
- e. At least how many CCRs are needed, to be able to take care of at least 1000 customer requests per hour, 99% of the time? [8]

4. The number of eggs laid by a bird, say N , has a $\text{Poisson}(\lambda)$ distribution, and let p denote the probability of a laid egg hatching into a chick. Also assume that the events of the laid eggs hatching into chicks are independent of one another. Let X denote the number of chicks thus born to a bird. Answer the following:

- a. Write down the joint p.m.f. of (X, N) . [2]
- b. Show that marginally $X \sim \text{Poisson}(\lambda p)$. [5]
- c. For a certain species of bird, $\lambda = 5$ and $p = 0.8$. Answer the following:
 - i. What is the probability of a bird of this species laying 4 eggs and 3 of them hatching into chicks? [6]
 - ii. Find the median number of chicks born to a bird of this species. [2]
- d. Find the regression of X on N , the correlation coefficient between them, and interpret these two quantities (for an arbitrary λ and p). [1+5+3=9]
- e. Show that the conditional distribution of $N|X$ is $X + \text{Poisson}(\lambda q)$, where $q = 1 - p$. [4]
- f. Again consider a bird of the same species as in c, which is known to have 3 chicks. Predict the value of the number of eggs it had laid, using:
 - i. the predictor with minimum mean squared error of prediction, and [2]
 - ii. the most likely value. [2]

5. A telecom company seeks online applications to fill 50 vacancies for tech-support. An application is deemed to be valid, if the applicant has a technical degree and at least two years of relevant experience. 70% of the applicants have a technical degree, while only 60% have at least two years of relevant experience. A quarter of the applicants, who have less than two years of relevant experience, do not have a technical degree either. Answer the following:

- a. What percentage of the applications is valid? [5]
- b. How many applications is one expected to go through one by one, to find 50 valid ones? What is the most likely value of this number? [2+2=4]
- c. At most how many applications one has to go through one by one, so that one can be 90% certain of finding 50 valid applications among them? [4]
- d. What is the probability of finding exactly 50 valid applications, in a bunch of 125 such applications? [5]
- e. What common assumptions are being made for answering b, c and d above? [2]